

Linear Relations

Refer to Section 4.2

Glossary Terms:

- **Linear Relation:** a relation/equation that has a straight-line graph
- **X axis:** the horizontal axis on a graph
- **Y axis:** the vertical axis on a graph

Notes:

In this section we will learn to associate an equation with a table of values, and from this, we will be able to create a graph.

We'll start off by creating some tables of values from the following equations.

$$y = 2x$$

x	y
1	2
2	4
3	6
4	8
5	10

$$y = 2x + 1$$

x	y
1	3
2	5
3	7
4	9
5	11

Word Problems that involve linear relations

A phone service provider charges a new customer \$20 for a new phone and 10¢ per minute of talk time. Create a table of values for a customer based on 50, 100, 150 & 200 minutes of talking.

Let C = total cost $(= 0.1(m) + 20)$

Let M = mins of talk

4.2

Linear Relations

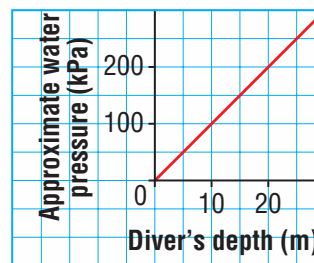
FOCUS

- Analyze the graph of a linear relation.

When a scuba diver goes under water, the weight of the water exerts pressure on the diver.

Diver's Depth (m)	Approximate Water Pressure (kiloPascals)
0	0
5	50
10	100
15	150
20	200

Pressure on a Diver



What patterns do you see in the table and in the graph?
What do these patterns tell you about the relationship between depth and water pressure?

Investigate



A local phone company offers a cell phone plan that has a fixed cost per month and a cost related to the number of text messages sent. The fixed cost is \$20 and each message sent costs 10¢.

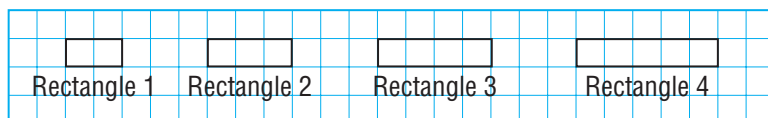
Represent the relation between the total cost and the number of text messages sent, as many different ways as you can.

Reflect & Share

Compare your representations with those of another pair of students. Did you use the same way to represent the pattern? If your patterns are different, explain your pattern to the other students.
If you represented the relation in a different way from your classmates, explain your way to them.

Connect

The first 4 rectangles in a pattern are shown below. The pattern continues.
Each small square has side length 1 cm.



The perimeter of a rectangle is related to the rectangle number.
We can use words, a table, a graph, and an equation to represent this relationship.
Each representation tells us about the relationship between the rectangle number and its perimeter.

In Words

Rectangle 1 has perimeter 6 cm; then, as the rectangle number increases by 1, its perimeter increases by 2 cm.

In a Table

	Rectangle Number, n	Perimeter, P (cm)	
	1	$6 = 2(1) + 4$	
+1	2	$8 = 2(2) + 4$	+2
+1	3	$10 = 2(3) + 4$	+2
+1	4	$12 = 2(4) + 4$	+2

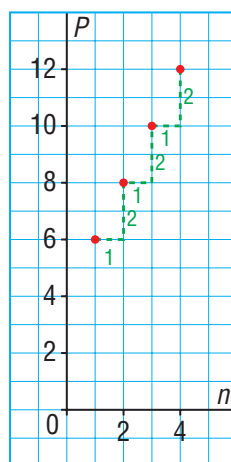
As the rectangle number increases by 1, the perimeter increases by 2 cm.

In a Graph

The graph also shows the pattern. After the first point, each point on the graph is 1 unit right and 2 units up from the preceding point. If we place a transparent ruler on the points, we see that they lie on a straight line.

We do not join the points because the data are discrete.

Graph of P against n



In an Equation

For rectangle n , the perimeter will be $2n + 4$.

The equation is: $P = 2n + 4$

The equation tells us that we can calculate the perimeter of any rectangle in the pattern by multiplying the rectangle number by 2, then adding 4.

The value of the variable P *depends* on the value of the variable n .

We say that P is the **dependent variable** and we plot it on the vertical axis.

The **independent variable** n is plotted on the horizontal axis.

When two variables are related, we have a **relation**.

Linear Relation

When the graph of the relation is a straight line, we have a **linear relation**.

In a linear relation, a constant change in one quantity produces a constant change in the related quantity.

In the relation above, a constant change of 1 in n produced a constant change of 2 cm in P .

Here is the equation of a linear relation: $y = 3x - 5$

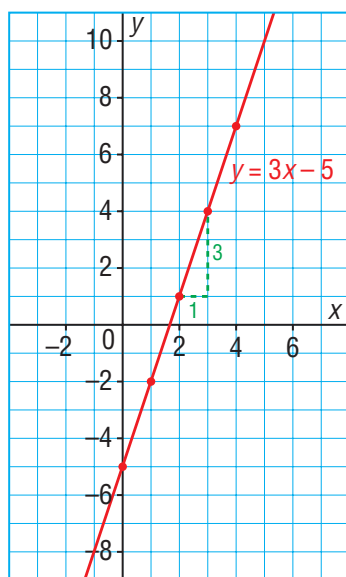
x is the independent variable and it is plotted on the horizontal axis.

y is the dependent variable and it is plotted on the vertical axis.

Here are the table and graph that represent this equation.

x	y
0	-5
1	-2
2	1
3	4
4	7

Diagram illustrating the constant change in x (+1) and the constant change in y (+3) between consecutive points.



Write the equation on the grid.

When x increases by 1, y increases by 3. This is shown in the table and on the graph.

Since the points lie on a straight line, the equation $y = 3x - 5$ represents a linear relation.

Since we are not told that the data are discrete, we join the points with a line.

Example 1**Graphing a Linear Relation from a Table of Values**

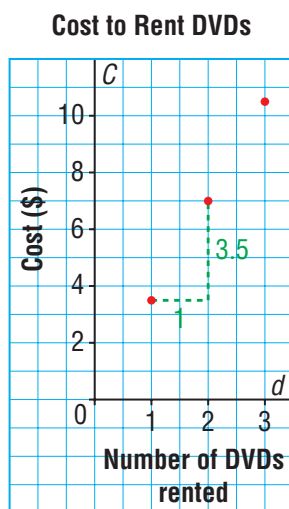
The table of values shows the cost of renting DVDs at an online store.

- Graph the data. Does it make sense to join the points on the graph? Explain.
- Is the relation linear? Justify your answer.
- Use the table to describe the pattern in the rental costs. How is this pattern shown in the graph?

Number of DVDs Rented, d	Cost, C (\$)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50

A Solution

- Plot the points on a grid.



Since the cost depends on the number of DVDs rented, plot d horizontally and C vertically.

The number of DVDs rented is a whole number. We cannot rent 1.5 DVDs or any other fractional number of DVDs. So, it does not make sense to join the points.

- The points on the graph lie on a straight line, so the relation is linear.
- As the number of DVDs rented increases by 1, the rental cost increases by \$3.50. Each point on the graph is 1 unit right and 3.5 units up from the previous point. The pattern of increases in the table produces a graph that is a straight line.

Example 2**Graphing a Linear Relation from an Equation**

A relation has the equation: $y = 6 - 3x$

- Create a table of values for the relation for values of x from -3 to 3 .
- Graph the relation. Does it make sense to join the points on the graph? Explain.
- What patterns are in the graph? How do these patterns relate to the table of values?
- Is the relation linear? Justify your answer.

Solutions

Method 1

- a), b) To create a table of values, substitute the given values of x in the equation:

$$y = 6 - 3x$$

Substitute: $x = -3$ Substitute: $x = -2$

$$\begin{aligned} y &= 6 - 3(-3) & y &= 6 - 3(-2) \\ &= 6 + 9 & &= 6 + 6 \\ &= 15 & &= 12 \end{aligned}$$

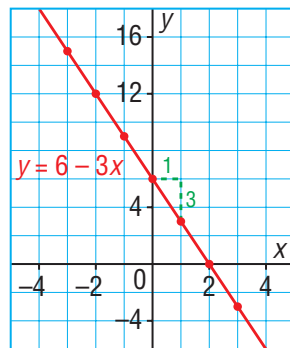
Substitute: $x = -1$ Substitute: $x = 0$

$$\begin{aligned} y &= 6 - 3(-1) & y &= 6 - 3(0) \\ &= 6 + 3 & &= 6 - 0 \\ &= 9 & &= 6 \end{aligned}$$

Use mental math to repeat the above process for $x = 1$, $x = 2$, and $x = 3$.

Write the values of x and y in a table.

x	y
-3	15
-2	12
-1	9
0	6
1	3
2	0
3	-3



Since the data are not discrete, join the points to form a line.

- c) On the graph, to get from one point to the next, move 1 unit right and 3 units down.
In the table, when x increases by 1, y decreases by 3.
- d) The relation is linear because its graph is a straight line.

Method 2

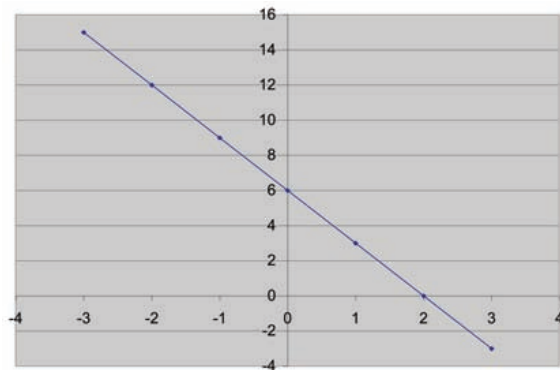
Use a spreadsheet.

- a) Input the equation and make a table.

	A	B
1	x	y
2	-3	15
3	-2	12
4	-1	9
5	0	6
6	1	3
7	2	0
8	3	-3

- b) Highlight the table.

Graph the data.



Example 3**Solving Problems Using a Linear Relation**

The student council is planning to hold a dance. The profit in dollars is 4 times the number of students who attend, minus \$200 for the cost of the music.

- Write an equation that relates the profit to the number of students who attend.
- Create a table of values for this relation.
- Graph the data in the table. Does it make sense to join the points? Explain.
- How many students have to attend to make a profit?

A Solution

- a) Profit in dollars = $4 \times \text{number of students who attend} - 200$

Choose variables to represent the numbers that change.

Let n represent the number of students who attend.

Let P represent the profit in dollars.

An equation is: $P = 4n - 200$

- b) Choose 3 values for n , then calculate the corresponding values of P .

Use the equation: $P = 4n - 200$

Substitute: $n = 0$

$$\begin{aligned} P &= 4(0) - 200 \\ &= 0 - 200 \\ &= -200 \end{aligned}$$

Substitute: $n = 50$

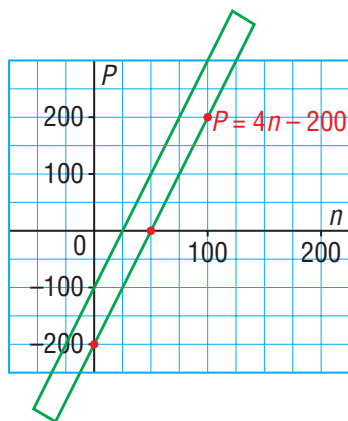
$$\begin{aligned} P &= 4(50) - 200 \\ &= 200 - 200 \\ &= 0 \end{aligned}$$

Substitute: $n = 100$

$$\begin{aligned} P &= 4(100) - 200 \\ &= 400 - 200 \\ &= 200 \end{aligned}$$

n	P
0	-200
50	0
100	200

- c) Plot the points on a grid.



A straightedge verifies that the points lie on a straight line.

Some values between the plotted points are permitted, but not others.

For example, there could be 82 students attending the dance, but not 82.5.

So, the points are not joined.

- d) When P is negative, a loss is made.

When $P = 0$, $n = 50$, and the profit is 0.

When $P > 0$, $n > 50$, and there is a profit.

So, 51 or more students have to attend before a profit can be made.

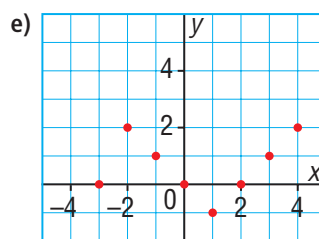
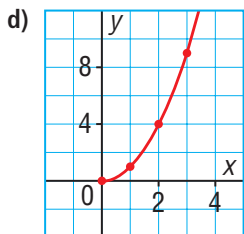
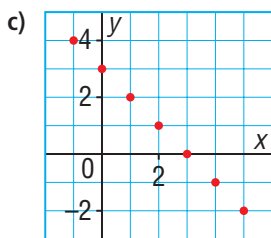
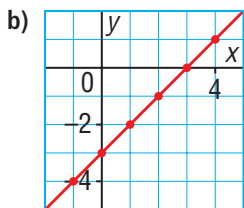
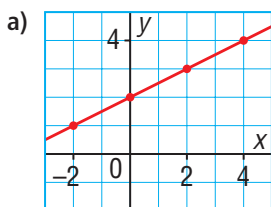
Discuss the ideas

- a) How do you know whether a graph represents a linear relation?
 - b) How do you know whether a table of values represents a linear relation?
- a) How many points do you need to graph a line?
 - b) Why do we often use 3 points? Should we use more points? Explain.
- How do you know when to connect the points on a graph?

Practice

Check

4. Which graphs represent a linear relation?
How do you know?



Apply

5. For each table of values below:
- Does it represent a linear relation?
 - If the relation is linear, describe it.
 - If the relation is not linear, explain how you know.

a)

x	y
1	4
2	13
3	22
4	31
5	40

b)

x	y
9	8
8	11
7	14
6	17
5	20

c)

x	y
0	0
1	2
2	6
3	12
4	20

d)

x	y
1	3
4	5
7	7
10	9
13	11

6. Graph the linear relations you identified in question 5. How does each graph verify your answers to question 5?

7. Copy and complete each table of values.

a) $y = 2x$

x	y
1	2
2	4
3	6
4	8

b) $y = x + 2$

x	y
1	3
2	4
3	5
4	6

c) $y = -2x$

x	y
2	-4
4	-8
6	-12
8	-16

d) $y = x - 2$

x	y
4	2
5	3
6	4
7	5

8. Here is a partially completed table of values for a linear relation. $y = 3x$

x	2	3	4	5	6	7	8
y	6	9	12	15	18	21	24

- Determine the missing values of y . Explain how you found these values.
- Describe the patterns in the table.
- Write an equation that represents the linear relation. How do you know that your equation is correct?
- Graph the data. How are the patterns you described in part b shown in the graph?
- Suppose you want to determine the value of y when $x = -1$. How could you use the table and equation to do this? What is the value of y when $x = -1$?

9. Each table of values represents a linear relation. Copy and complete each table. Explain your reasoning.

a)

x	y
2	11
3	14
4	17
5	20
6	23

b)

x	y
1	7
3	8
5	9
7	10
9	11

c)

x	y
-4	11
-2	7
0	3
2	7
4	11

d)

x	y
4	-10
6	-7
8	-4
10	-1
12	2

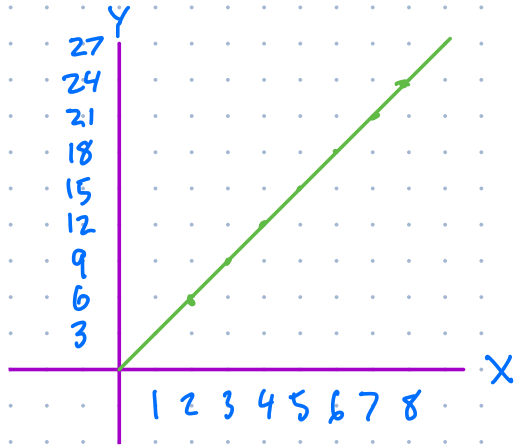
10. Create a table of values for each linear relation, then graph the relation.

Use values of x from -2 to 2 .

- $y = 3x$
- $y = x + 3$
- $y = x - 3$
- $y = 5 - x$
- $y = 1 - 4x$
- $y = -2x - 3$

11. Jin is cycling at an average speed of 4 m/s. He travels a distance, d metres, in t seconds.
- Write an equation that relates d and t .
 - Create a table of values for this relation.
 - Graph the data. Should you join the points? Explain your reasoning.
 - Is the relation between distance and time linear?
 - How do you know from the table of values?
 - How you know from the graph?
 - How far does Jin travel in 3.5 h?
 - What time does it take Jin to travel 17 km?

Pg. 171 (4.2) question 8.



$$5 + 3$$

$$5 - 0$$

18:

a)

x	y
-2	-6
-1	-3
0	0
1	3
2	6

b)

x	y
-2	1
-1	2
0	3
1	4
2	5

c)

x	y
-2	-5
-1	-4
0	-3
1	-2
2	-1

d)

x	y
-2	7
-1	6
0	5
1	4
2	3

e)

x	y
-2	9
-1	5
0	1
1	-5
2	-9

f)

x	y
-2	1
-1	-1
0	-3
1	-5
2	-7



f)

y

x



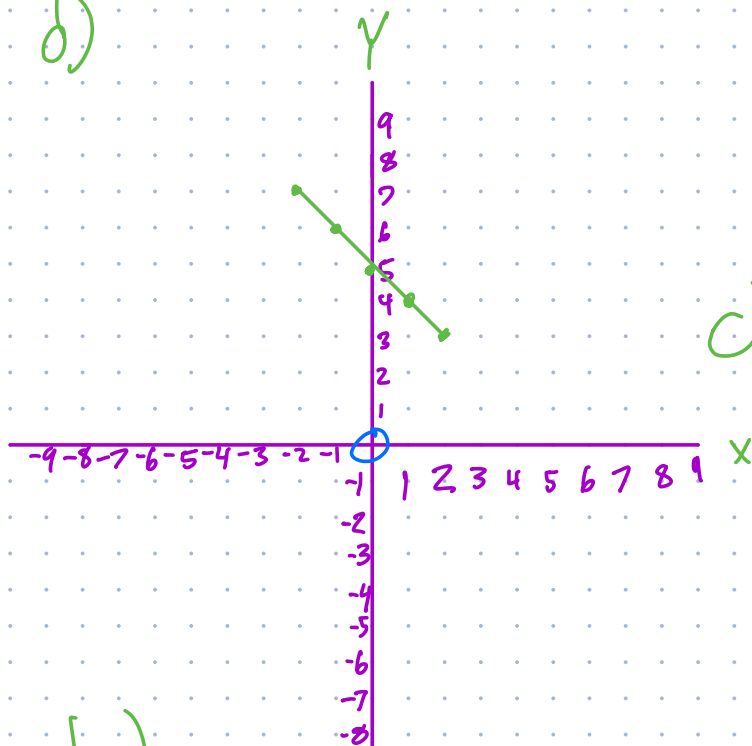
e)

y

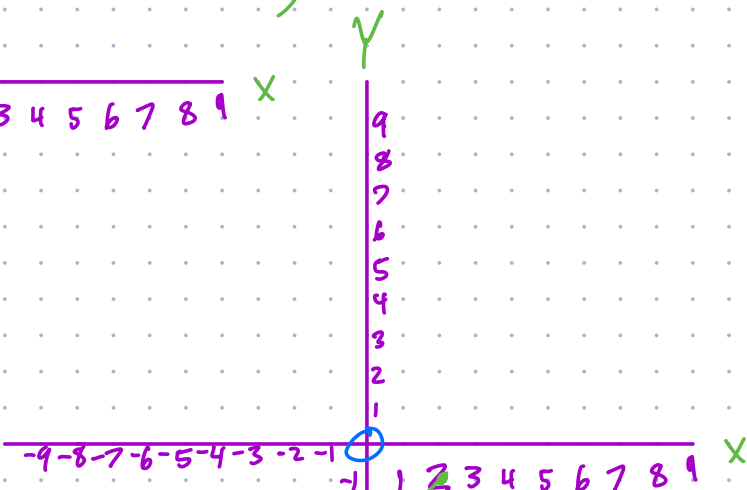


x

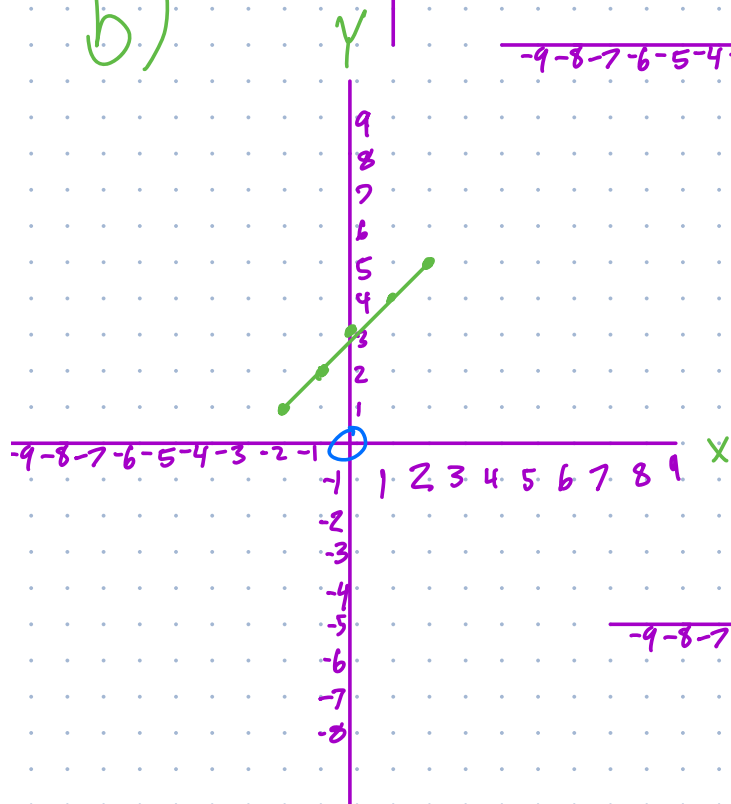
b)



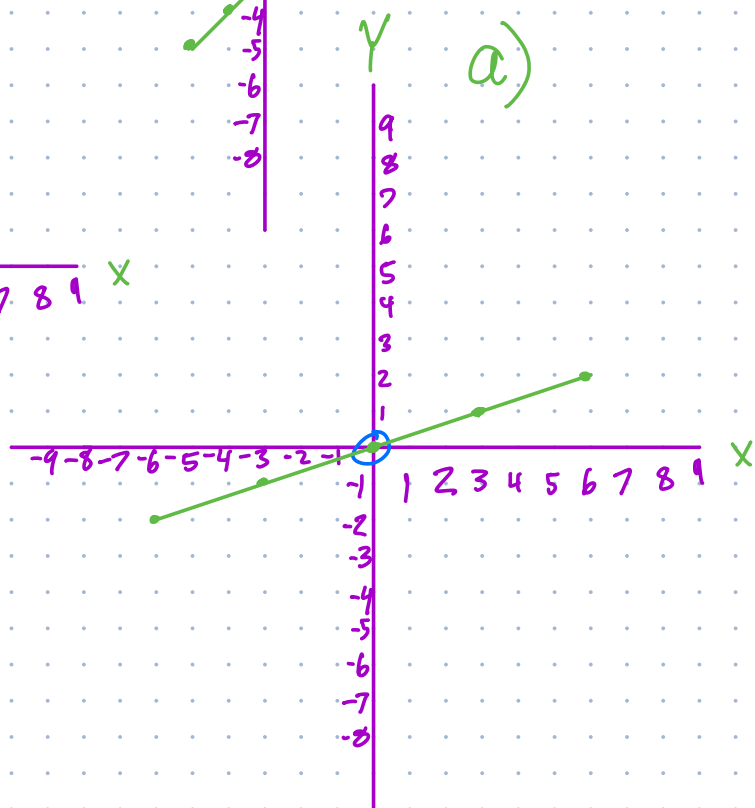
c)



b)



a)



12. In 2008, the Goods and Services Tax (GST) was 5%. To determine the tax, T dollars, charged on a given purchase price, p dollars, multiply the purchase price by 0.05.

- Write an equation that relates T to p .
- Copy and complete this table of values.

p	0	10	20	30	40
T					



- What patterns do you see in the table?
- Graph the data.
Which variable will you plot on the horizontal axis? Explain your reasoning.
- Should you connect the points on the graph? Explain.
- How are the patterns in the table shown in the graph?

13. An amusement park charges an admission fee of \$10, plus \$2 per ride.
- Choose variables to represent the total cost in dollars and the number of rides that are taken. Write an equation that relates the total cost to the number of rides.
 - Graph the equation.
 - What is the total cost for 7 rides?
 - How many rides can be taken for a total cost of \$38?

14. **Assessment Focus** Danica is having a party. She estimates that she will need 3 pieces of pizza for each guest invited, and 6 extra pieces in case someone shows up unexpectedly.

- Explain why this situation can be represented by the equation $P = 3n + 6$. What do P and n represent in the equation?
- Make a table of values for the relation.
- Graph the data. Will you join the points on the graph? Explain.
- Is the relation linear?
 - How do you know from the table of values?
 - How do you know from the graph?
- If the relation is linear, explain what this means in the context of this situation.

15. A small plane is at a height of 1800 m when it starts descending to land. The plane's height changes at an average rate of 150 m per minute.

- Choose variables to represent the height in metres and the time in minutes since the plane began its descent. Write an equation that relates the height to the time.
- Graph the equation.
- What is the height of the plane 6 min after it began its descent?
- When is the plane 100 m above the ground?



16. Jada rollerblades from Regina to Saskatoon to raise funds for cancer research. The trip is 250 km. Jada estimates that she can rollerblade at an average speed of 8 km/h.



- a) Choose variables to represent the time Jada has travelled in hours and the distance in kilometres that she has yet to travel. Write an equation that relates the distance to the time.

- b) Graph the equation.
c) How far has Jada still to travel after 12 h?
d) How many hours will it take Jada to complete the trip?

17. Describe a situation that could be represented by each equation.

- a) $M = 2n + 5$ b) $E = 3.50n$
c) $C = 12 + 5d$ d) $H = 100 - 5n$

Take It Further

18. This table of values represents a linear relation. Copy and complete the table. Explain your reasoning.

x	-3	-1	2	5	9	14	20
y	29		23				

Reflect

What does it mean when we say that the relation between two quantities is linear? What patterns are there in the table of values and in the graph of a linear relation? Include examples in your explanation.

Math Link

Science

When an object falls to the ground, it accelerates due to the force of gravity. The relation between the speed of the object and the time it falls is linear.

