Question:
How can we test the pythagorean theorem and the law of reflection?

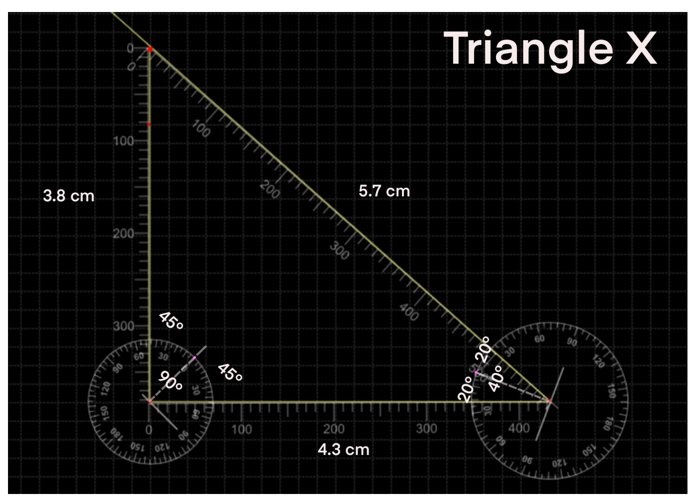
Hypothesis:
With the assistance of mirrors, light sources, protractors, and various other tools; we could test the pythagorean theorem and law of reflection by creating multiple laser displays.

Materials:
- paper
- pencil
- protractor
- ruler
- laser
- mirrors

OR a digital resource such as:
- https://ricktu288.github.io/ray-optics/simulator_old/

Procedure:
1. Open https://ricktu288.github.io/ray-optics/simulator_old/ OR grab materials.
2. Set up a "single ray" (top left).
3. Use a "line segment" mirror to reflect the ray into a 90 degree angle.
4. Then, use another line segment mirror to reflect the ray to make a closed right triangle.
5. Use protractors and rulers to measure the angles and confirm the law of reflection along with the pythagorean theorem.
6. Using the rulers, check the length of the legs. Use the formulas $a^2 + b^2 = c^2$ and $c^2 - b^2 = a^2$ to find the hypotenuse or leg length and test the pythagorean theorem.
7. Use a ruler to measure the hypotenuse of the triangle and confirm the formula was correct.
8. Use the formula $a \div 2 = b$ to calculate the measurements of the angle of incidence and angle of reflection. Then, use the protractors to find the distance of the angles from the "normal."
9. Repeat 2-3 times for more data, then write out data and results.

Data/Results:
The legs of triangle X were 3.8 cm and 4.3 cm.



4.3^2 cm + 3.8^2 cm =
18.49 cm + 14.44 cm = 32.93 cm
And the square root of 32.93 cm = about 5.7 cm
The hypotenuse is about 5.7 cm.

OR if these equations were to be displayed algebraically:

The triangle angles that were reflected by the mirrors were 90° and 40°. The law of reflection states that the angle of incidence and angle of reflection should be the same distance from the normal.

90° ÷ 2 = 45°

$40° \div 2 = 20°$

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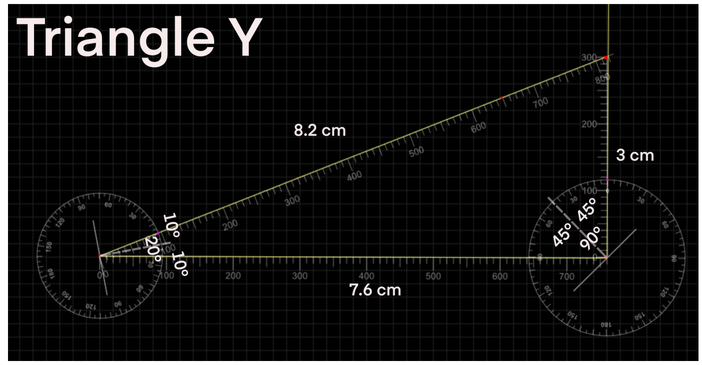
The legs of triangle Y were 3 cm and 7.6 cm.



3^2 cm + 7.6^2 cm =
9 cm + 57.76 cm = 66.76 cm
And the square root of 66.76 cm = about 8.2 cm.
The hypotenuse is about 8.2 cm.

The triangle angles that were reflected by the mirrors were 20° and 90°

$20° \div 2 = 10°$

$90° \div 2 = 45°$

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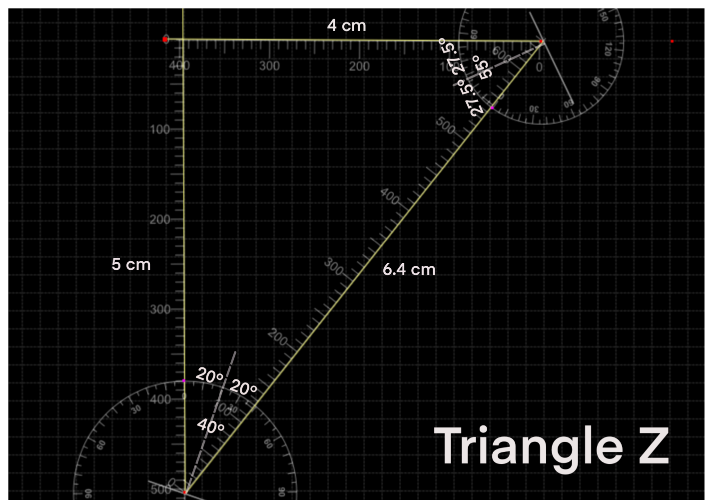
The legs of triangle Z were 4 cm and 5 cm.



4^2 cm + 5^2 cm =
16 cm + 25 cm = 41 cm
And the square root of 41 = about 6.4 cm.
The hypotenuse is about 6.4 cm.

The triangle angles that were reflected by the mirrors were 40° and 55°

$40° \div 2 = 20°$

$55° \div 2 = 27.5°$

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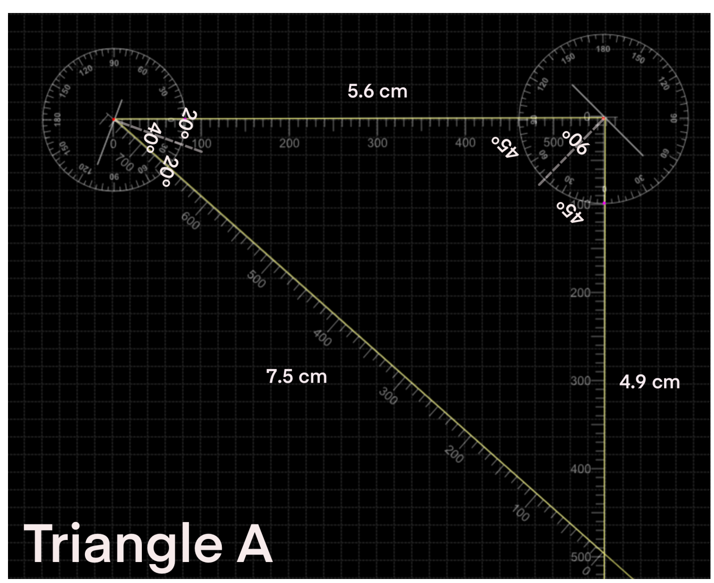
For triangle A, I had decided to calculate a leg length instead of the hypotenuse. The hypotenuse and upper leg were 7.5 cm and 5.6 cm.



7.5^2 cm - 5.6^2 cm =
56.25 cm - 31.36 cm = 24.89 cm
The square root of 24.89 = about 4.9 cm
The triangle leg is about 4.9 cm.

The triangle angles that were reflected by the mirrors were 20° and 90°

$20° \div 2 = 10°$

$90° \div 2 = 45°$
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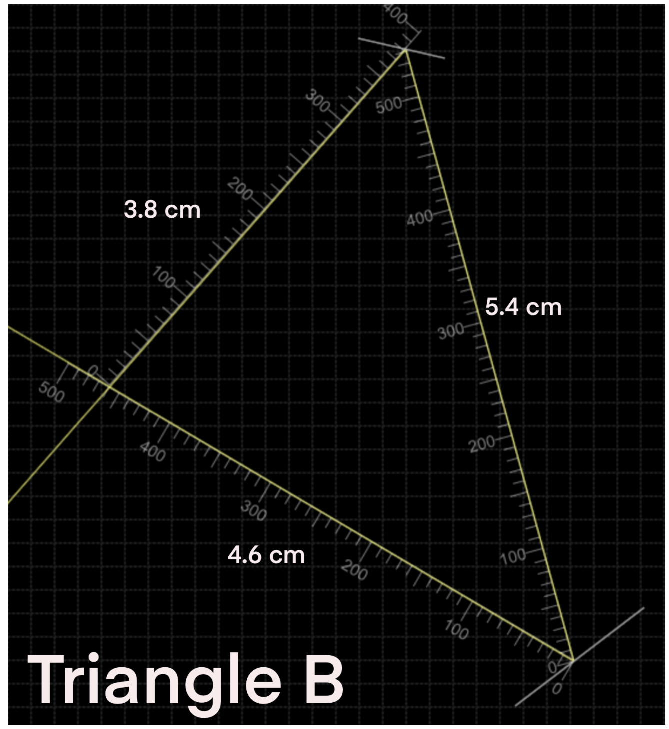
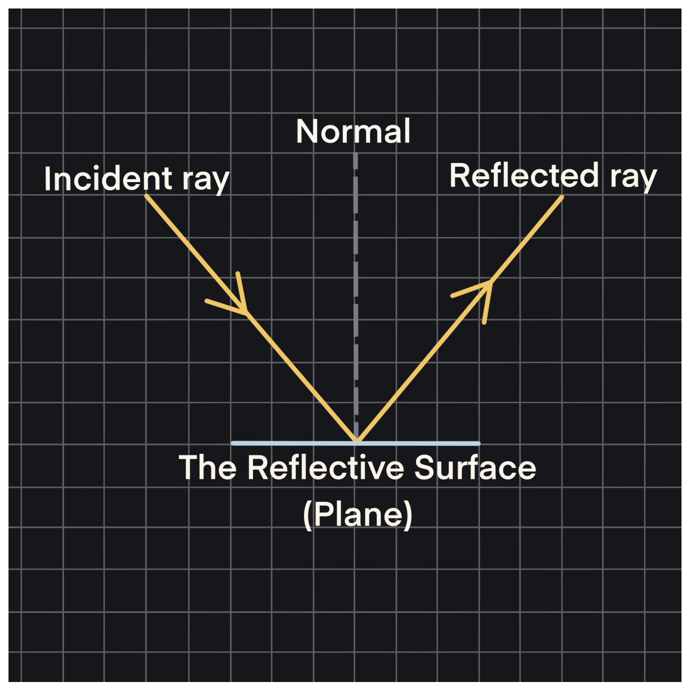
For triangle B, I decided to test the pythagorean theorem by using a scalene triangle rather than a right triangle.



5.4^2 cm - 3.8^2 cm =
29.16 cm - 14.44 cm = 14.72 cm
The square root of 14.72 cm = about 3.8 cm
However, when I checked the measurement of the remaining triangle side, it was 4.6 cm. This means that the pythagorean theorem does not work on scalene triangles.

Conclusion:

My hypothesis was correct, as I was able to test both the pythagorean theorem and law of reflection using the aforementioned materials. I first tested the pythagorean theorem by first using the formula $a^2 + b^2 = c^2$ which then gave me the following hypotenuse measurements:



Triangle X: 5.7 cm
Triangle Y: 8.2 cm
Triangle Z: 6.4 cm

In the formula I used to calculate the length of the hypotenuse of all triangles, the letters A and B represent the legs of the right triangle. The letter C represents the hypotenuse of the triangle. Essentially, the pythagorean theorem explains that the measurement of the legs of a right triangle squared will equal the squared number of the hypotenuse's measurement. You can then use a calculator to find the square root of that number, which will be the hypotenuse's measurement, which is part of what I did for this experiment.

However, for triangle A, I made a few changes to the formula. I instead used the formula $c^2 - b^2 = a^2$ to calculate the measurement of a leg instead of the hypotenuse. Using that formula, I was then able to calculate the measurement of one of the legs which was 4.9 cm.

I had tried using the formula $c^2 - b^2 = a^2$ on triangle B, which was a scalene triangle. However, this formula did not work to calculate the remaining triangle side. The calculation according to the formula was 3.8 cm but the triangle side was 4.6 cmThis means the pythagorean theorem does not work on scalene triangles.

In order to test the law of reflection, I then measured the angles that were made by the mirrors through the process of reflecting light. I had used the formula $a \div 2 = b$ to calculate the measurements of the angle of incidence and angle of reflection as seen below. As stated by the law of reflection, the normal is a line of which is perpendicular to a surface, which in this case, is a mirror. The angle of incidence is the angle that takes place between the incident ray and the normal. The angle of reflection is the angle that takes place between the reflect ray and the normal.

Triangle X:
$90^\circ \div 2 = 45^\circ$

$40^\circ \div 2 = 20^\circ$

Triangle Y:
$20^\circ \div 2 = 10^\circ$

$90^\circ \div 2 = 45^\circ$

Triangle Z:
$40^\circ \div 2 = 20^\circ$

$55^\circ \div 2 = 27.5^\circ$

Triangle A:
$90^\circ \div 2 = 45^\circ$

$40^\circ \div 2 = 20^\circ$

Afterwards, I checked the protractors and rulers to ensure my math was correct, which it was. Throughout my experiment, I did have to replace the rulers multiple times because I hadn't properly lined them up, causing my math to be incorrect. However, after realigning the rulers, I was able to successfully continue. This was an interesting experiment and I look forward to the next milestone!